Validation of a Computational Aero-Acoustics formulation based on Lighthill’s analogy for a cooling fan and mower blade noise

Methodology implemented in FluidConnection-AcuSolve-Actran/LA

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Process overview

- Two models:
  1. CFD for source generation
  2. Variational Lighthill’s analogy for acoustic wave propagation
Process in more detail

- **Core components:**
  - CAD (Pro/E or Catia V5)
  - FluidConnection
  - AcuSolve
  - Actran/LA
Equations

Navier-Stokes

\[ \frac{\partial u_i}{\partial t} + \nabla \cdot (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \tau_{ij} \quad \text{in} \ \Omega, \]
\[ \frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i u_j) = 0 \quad \text{in} \ \Omega, \]
\[ p(t = 0) = p_0 \quad \text{in} \ \Omega, \]
\[ u(t = 0) = u_0 \quad \text{in} \ \Omega, \]
\[ u = u_{\text{inlet}} \text{ or } p = -\frac{1}{2} \rho (u \cdot n)^2 \quad \text{on} \ \Gamma_{\text{inlet}}, \]
\[ \tau \cdot n = 0 \text{ and } p = 0 \quad \text{on} \ \Gamma_{\text{outlet}}, \]
\[ u = u_{\text{wall}} \quad \text{on} \ \Gamma_{\text{wall}}, \]

where \( \Gamma = \partial \Omega, \ \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \delta_{ij} \lambda \nabla \cdot u, \ p_0 \text{ and } u_0 \text{ are initial conditions, and } f \text{ is a body force.} \)

If \( \Gamma_{\text{inlet}} = \Gamma_{\text{outlet}} = 0, \) the pressure \( p \) is set to zero in a node away from source regions.

Wave equation

\[ \frac{\partial^2 \psi}{\partial t^2} - \frac{a_0^2}{\frac{\partial^2 \psi}{\partial x_i x_i}} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \]

\[ T_{ij} = \rho u_i u_j + ((p - p_0) - \frac{a^2}{2} (v - v_0)) \delta_{ij} - \tau_{ij} \]

\[ \frac{\partial^2 \varphi}{\partial t^2} - \frac{a_0^2}{\frac{\partial^2 \varphi}{\partial x_i x_i}} = \frac{\partial^2 \psi}{\partial x_i \partial x_j} \]
Variational formulation

\[ \int_\Omega \left( \frac{\partial^2 L_a}{\partial t^2} \varphi + a_0^2 \frac{\partial L_a}{\partial x_i} \frac{\partial \varphi}{\partial x_i} \right) d\Omega = -\int_\Omega \frac{\partial T_{ij}}{\partial x_j} \frac{\partial \varphi}{\partial x_i} d\Omega + \int_\Gamma \frac{\partial \Sigma_{ij}}{\partial x_j} n_i \varphi d\Gamma \quad \forall \varphi \]

\[ T_{ij} = \varrho u_i u_j + ((p - p_0) - a_0^2 (\varepsilon - \varepsilon_0)) \delta_{ij} - \tau_{ij} \]

Volumetric sources

\[ \Sigma_{ij} = \varrho u_i u_j + (p - p_0) \delta_{ij} - \tau_{ij} \]

Surface sources
Finite Element Method

- CFD mesh
- Acoustic mesh
- Integrated element
- Overlap in CFD code

Both meshes imported in CFD code

Direct integration of source terms

✓ Small error (no projection error)
Fan example
Computed noise sources

- Iso-surface of magnitude of Lighthill’s tensor

\[ T_{ij} = \epsilon u_i u_j + \left( \rho - \rho_0 \right) \alpha_0^2 \left( \rho - \rho_0 \right) \delta_{ij} - \tau_{ij} \]
Acoustic propagation

200 Hz

600 Hz
Acoustic SPL spectrum

• Evaluate sound pressure level (SPL) in virtual microphones

• Far and near field

• Good comparisons with experiments for the most important frequency range in the far field
Mower example

- Noise sources from CFD
Acoustic propagation

100 Hz

1000 Hz
Acoustic SPL spectrum
Value in the design process

- Transient CFD simulation
  - Flow characteristics, separation etc.
  - Noise source study
- Acoustics propagation simulation
  - Sound pressure level map
  - Phase information

2-3 weeks simulation on 24 CPUs Cray XD1 cluster
(20M elements 10K time steps)

2-3 days simulation on 4 CPUs Cray XD1 cluster
(1M elements 1K frequencies)
Conclusion

• Integrated approach where the source terms used for Actran/LA are computed exactly within AcuSolve. Errors in simulations kept under control. This feature is unique to the AcuSolve - Actran/LA combination

• Valuable output both from CFD animations and acoustic sound pressure fields

• Sound pressure level compares well with experiments